PH4214: Research Project II

Presentations

Abhay Saxena(21MS086)

Under guidance of Dr. Kamaraju Natarajan

Cyclotron Resonance Spectroscopy in a High Mobility Two-Dimensional Electron Gas Using Characteristic Matrix Methods

David J. Hilton

(Department of Physics, The University of Alabama at Birmingham, Birmingham, AL)

Presentation by: Abhay Saxena

March 18, 2025

Reference: [Hilton, 2012]

Schema

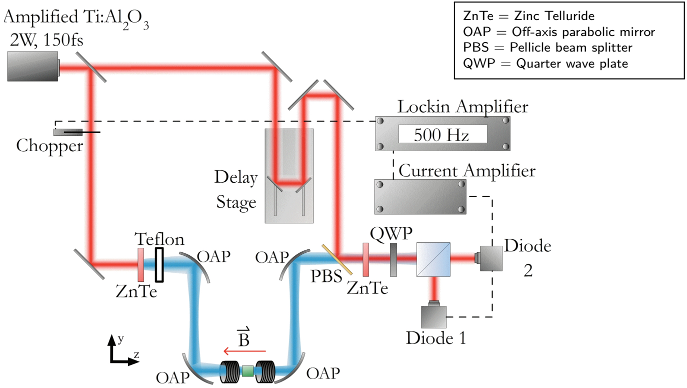


Figure: A diagram of a typical terahertz time-domain spectrometer with an external magnetic field in the Faraday geometry ($\vec{B} \perp 2DEG$). There exist four quartz windows on either the entrance or exit of the split coil magnet, which generate satellite pulses in addition to the gallium arsenide 2DEG/Substrate. The $\hat{y} - \hat{z}$ 2DEG sample coordinate system, as defined in the manuscript, is indicated in this diagram. The polarization of the input terahertz pulse is perpendicular to the page ($\hat{x}$), while the transmitted pulse field is polarized in the $\hat{x} - \hat{y}$ plane.

Basic Process

- **Pulse generation:** Electric field of the 100fs pulse as a Gaussian-modulated cosine is generated by Ti:Sapphire laser.
- **Optical rectification:** THz polarization is parallel to the incoming laser beam's polarization, generating a THz pulse due to second order non-linearity by optical rectification in a ZnTe crystal, ***linearly polarized say along*** $\hat{\mathbf{x}}$.
- Under proper phase matching, results in the emission of a coherent broadband spectrum with a bandwidth of approximately $\Delta\nu = 0.8THz = \frac{1}{2\pi\sigma} = \frac{c}{2\pi d}(n_c - 1)$ that is centered near $\nu_0 = 0.5THz$ and has $|E_0| \sim 10kV/m$.
- Can be modeled using a Gaussian-modulated sine pulse:

$$E(t) = E_0 \cdot \exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right) \cdot \sin\left(2\pi\nu_0(t-t_0) + \pi\right)$$

Basic Process

Induced by Landau quantization in 2DEG:

- **Circular dichroism** causes differential absorption or rotation of the left- and right-circularly polarized components of the light, which means that the polarization is affected by the material's magnetic-field-dependent properties (breaking of TRS).
- **Birefringence** (different refractive indices) causes a phase difference between the orthogonal polarization components of the pulse.
- hence, the transmitted pulse is elliptically polarized.

Basic Process

- **Linear electrooptic effect:** A THz pulse changes ZnTe crystal's refractive index: $\Delta n(t) = n_c \cdot r_{41} \cdot E(t)$, where r_{41} is the non-zero electro-optic tensor.
- ZnTe crystals are cut along the (110) plane.
- Effective Pockel's response is maximized when THz is polarized along [001].
- Probe beam is at 45°, to the principal axes of the (110) plane.
- For our setup, [001] is aligned along $\hat{y}$ for maximum Pockel's response.
- We measure this component for both $\pm B_{ext}$ to ensure proper alignment (100:1 polarization extinction ratio).

THz Time Domain Waveforms

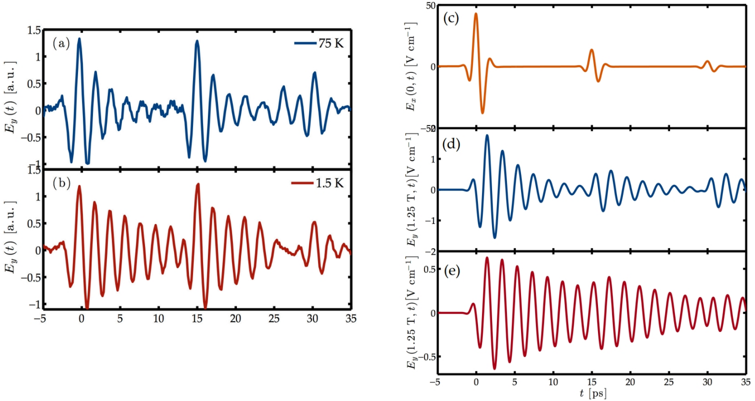


Figure: Terahertz time-domain waveforms taken in the high mobility 2DEG, E(t) for different Decay lifetimes T_2 in $B_{ext} = 1.25T$. a and b are experimental data. c, d and e are simulations for E_x, E_y at $T_2 = 0.5ps$ and E_y at $T_2 = 15.1ps$ respectively. (best fit for a and b).

Coherence in 2DEG

- B_{ext} is aligned in z-direction, perpedicular to 2DEG, causing continuous energy spectrum of free electrons quantizes into discrete energy levels known as Landau levels (LLs).

$$E_n = \hbar\omega_c(n + \frac{1}{2})$$

- Energy spacing, $\Delta E = \hbar eB/m^*$ a cyclotron frequency $\omega_c = \frac{eB_{ext}}{m^*}$.
- THz is tuned to resonance with Landau energy spacing between highest filled $|n\rangle$ and $|n+1\rangle$ levels.
- Results in formation of coherent superposition of states.

Coherence in 2DEG

- **Superposition of states:** A and B are complex coefficients that evolve over time with well defined relative phase of w_c (coherence).

$$|\psi(t)\rangle = Ae^{-iE_n t/\hbar}|n\rangle + Be^{-iE_{n+1} t/\hbar}|n+1\rangle$$

- **Dephasing:** Due to Scattering events, Inhomogenity of B_{ext}, and finite temperature, coherence is lost over time. Decay lifetime T_2 such that:

$$E_y(t) \propto e^{-t/T_2} cos(\omega_c t + \phi)$$

- Hence need Low temperature and high mobility.

Satellite Pulses

- finite thickness of the eight quartz magnet windows and gallium arsenide substrate lead to formation of a series of satellite terahertz pulses
- Delays determined by their thicknesses and the refractive index of quartz or GaAs.
- Etalon effect (Fabry-Pérot) due to thin-film interference. We take a subtrate with thickness 7.5 times wavelength of 1 thz.
- Small $\Delta t \approx 15 ps$ between the two pulses allows satellite pulse to overlap with tail of previous oscillating field, leading to interference.

Solving for Satellite Pulses

- **Introducing wedge:** primary and satellite pulses travel with different propagation vector directions, allowing seperation.
- **Windowing:** Crop time domain data behind satellite pulse before using fft. Limited by frequency resolution of acquired data.
- Need TMM would allow analysis of long cyclotron decoherence lifetimes
- Future application: Use of time-delayed satellite pulses is an experimental method of coherent control of the quantum superposition states.

Defining basis

Due to eliptical polarization, in $x - y$ plane of our original basis:

$$[\hat{x}, \hat{y}, \hat{z}]$$

We define a new basis:

$$[\hat{\sigma}_+, \hat{\sigma}_-, \hat{z}]$$

with,

$$\hat{\sigma_\pm} = \frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y})$$

And monochromatic components of the total electromagnetic field, polarized in the $x - y$ plane, can be written:

$$\begin{aligned} \vec{E}(z,\nu) &= U(z,\nu)\hat{\sigma}_+ + P(z,\nu)\hat{\sigma}_- \\ \vec{H}(z,\nu) &= V(z,\nu)\hat{\sigma}_+ - Q(z,\nu)\hat{\sigma}_- \end{aligned} \tag{1}$$

Anisotropic Materials

- **Anisotropic materials:** exhibit different properties in different directions like Birefringence and Circular dichroism.
- **Dielectric tensor:** $\bar{\varepsilon}$ describes anisotropic materials with broken TRS but no birefringence.

$$\bar{\varepsilon}(z,\nu) = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ -\varepsilon_{xy} & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \equiv \begin{bmatrix} \varepsilon_{pp} & 0 & 0 \\ 0 & \varepsilon_{mm} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \tag{2}$$

- $\varepsilon_{pp} = \varepsilon_{xx} + i\varepsilon_{xy}$ and $\varepsilon_{mm} = \varepsilon_{yy} - i\varepsilon_{xy}$.
- $\varepsilon_{pp} = \varepsilon_{mm}$ for isotropic materials with TRS, since, $\varepsilon_{xy} = 0$.
- $\bar{\mu}$ is diagonal matrix with all elements μ.

Maxwell equations

Faraday's and Ampere's laws in the frequency domain:

$$\begin{aligned}\nabla \times E(z,\nu) = &\quad +i2\pi\nu\mu(z,\nu)H(z,\nu) \\ \nabla \times H(z,\nu) = &\quad -i2\pi\nu\epsilon(z,\nu)E(z,\nu)\end{aligned} \tag{3}$$

This when applied to (1) gives:

$$\begin{aligned}\frac{dU}{dz} &= -2\pi\nu\mu\, V \\ \frac{dV}{dz} &= +2\pi\nu\epsilon\, U \\ \frac{dP}{dz} &= +2\pi\nu\mu\, Q \\ \frac{dQ}{dz} &= -2\pi\nu\epsilon\, P\end{aligned} \tag{4}$$

This shows U and V ($\hat{\sigma}_{+} component$) propogate independent of P and Q ($\hat{\sigma}_{-} component$).

Propogation

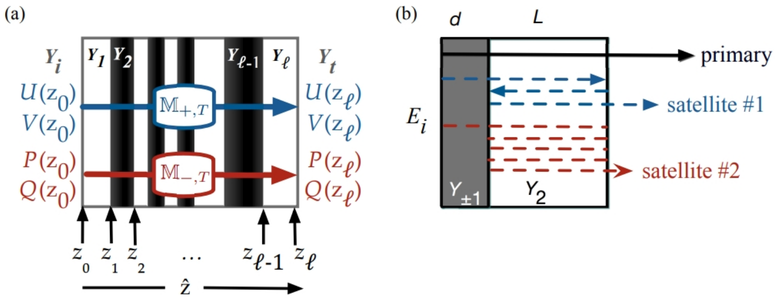


Figure: Propagation of a normally incident electromagnetic wave through an arbitrary stratified medium with l layers located at $z \in [z_0, z_1 \cdots z_{l-1}, z_l]$ with Admittance Y_j. 2DEG is Y_1 and GaAs substrate is Y_2.

Propogating Waves

For a layer $z_j \leq z \leq z_{j+1}$, the solution to the (4) is:

$$\begin{aligned}
P(z) &= P(z_j)\cos\left[\kappa_-(z-z_j)\right] + Q(z_j)\left\{+Y_-^{-1}\sin\left[\kappa_-(z-z_j)\right]\right\} \\
Q(z) &= Q(z_j)\cos\left[\kappa_-(z-z_j)\right] + P(z_j)\left\{-Y_-^{-1}\sin\left[\kappa_-(z-z_j)\right]\right\} \\
U(z) &= U(z_j)\cos\left[\kappa_+(z-z_j)\right] + V(z_j)\left\{-Y_+^{-1}\sin\left[\kappa_+(z-z_j)\right]\right\} \\
V(z) &= V(z_j)\cos\left[\kappa_+(z-z_j)\right] + U(z_j)\left\{+Y_+\sin\left[\kappa_+(z-z_j)\right]\right\}
\end{aligned} \tag{5}$$

Where Admittance is given by $Y_+ = \sqrt{\varepsilon_{pp}\mu^{-1}}$, $Y_- = \sqrt{\varepsilon_{mm}\mu^{-1}}$ and complex propogation vector magnitudes $\kappa_+ = 2\pi\nu\sqrt{\varepsilon_{pp}\mu}$, $\kappa_- = 2\pi\nu\sqrt{\varepsilon_{mm}\mu}$.

Characterstic Matrix Method

Lets write components of the electromagnetic wave in $\hat{\sigma}_{\pm}$:

$$Q_{+}(z,\nu) = \begin{bmatrix} U(z,\nu) \\ V(z,\nu) \end{bmatrix} \qquad Q_{-}(z,\nu) = \begin{bmatrix} P(z,\nu) \\ Q(z,\nu) \end{bmatrix} \tag{6}$$

If both $Q_{\pm}$ are known at z_j, the Q$\pm$ at $z_{j+1} = z_j + d$ can be found using,

$$Q_{\pm}(z_j,\nu) = \bar{M}_{\pm,j} Q_{\pm}(z_{j+1},\nu) \tag{7}$$

with propogation is described by

$$\bar{M}_{\pm,j} = \begin{bmatrix} \cos(\kappa_{\pm} d) & -Y_{\pm}^{-1} \sin(\kappa_{\pm} d) \\ -Y_{\pm} \sin(\kappa_{\pm} d) & \cos(\kappa_{\pm} d) \end{bmatrix} \tag{8}$$

Total Transfer matrix

We could define total transfer matrix $\bar{M}_{\pm,T}$ such that it is product of all individual transfer matrices:

$$\begin{aligned} \bar{M}_{\pm,T} &= \bar{M}_{\pm,1}\bar{M}_{\pm,2}\cdots\bar{M}_{\pm,l} \\ &= \begin{bmatrix} A_{\pm} & B_{\pm} \\ C_{\pm} & D_{\pm} \end{bmatrix} \end{aligned}$$

Where at $z < z_0$ before hitting interface is descibed by (1) with superposition of incident and reflected field.

Total Transfer matrix

We also get following relations for our two layer approximation as:

$$\begin{aligned} A_{\pm} = & \quad \sin\varphi_{\pm}\sin\theta - Y_{\pm,1}^{-1} Y_2 \cos\varphi_{\pm}\cos\theta \\ B_{\pm} = & \quad \pm\left[Y_2^{-1}\cos\theta\sin\varphi_{\pm} + Y_{\pm,1}^{-1}\cos\varphi_{\pm}\sin\theta \right] \\ C_{\pm} = & \quad \mp\left[Y_2\cos\theta\sin\varphi_{\pm} + Y_{\pm,1}\cos\varphi_{\pm}\sin\theta \right] \\ D_{\pm} = & \quad \sin\varphi_{\pm}\sin\theta - Y_{\pm,1} Y_2^{-1}\cos\varphi_{\pm}\cos\theta \end{aligned} \tag{9}$$

Where $\varphi_{\pm} = \kappa_{\pm,1} d$ and $\theta = \kappa_2 L$. Remember, $Y_+ = \sqrt{\varepsilon_{pp}\mu^{-1}}$, $Y_- = \sqrt{\varepsilon_{mm}\mu^{-1}}$ and $\kappa_+ = 2\pi\nu\sqrt{\varepsilon_{pp}\mu}$, $\kappa_- = 2\pi\nu\sqrt{\varepsilon_{mm}\mu}$.

Finding Permittivity

For cyclotron resonance-active polarization $\hat{\sigma}_+$, the susceptibility is

$$\bar{\chi}(\nu) = \chi_0 \frac{i - 2\pi(\nu - \nu_{CR})T_2}{1 + 4\pi^2(\nu - \nu_{CR})^2 T_2^2},$$

with imaginary and real parts describing circular dichroism and birefringence. The permittivity is

$$\varepsilon_{pp} = \varepsilon_b + (1 + \bar{\chi}(\nu))\varepsilon_0.$$

Selection rules enforce $\Delta m = 1$, allowing $\hat{\sigma}_+$ $(m = 1)$ to couple $|n\rangle \to |n+1\rangle$. $\hat{\sigma}_-$ $(m = -1)$ cannot, so

$$\varepsilon_{mm} = \varepsilon_b.$$

Transmission Matrix

$$t_+ \equiv \frac{U_t(z_\ell)}{U_i(z_0)} \tag{10a}$$

$$t_- \equiv \frac{P_t(z_\ell)}{P_i(z_0)} \tag{10b}$$

Boundary Conditions:

- At input: $E(z_0) = E_i + E_r$, $H(z_0) = Y_i(E_i - E_r)$.
- At output: $E(z_\ell) = E_t$, $H(z_\ell) = Y_t E_t$.

Using matrix propagation, the field relations yield:

$$\begin{bmatrix} E_i + E_r \\ Y_i(E_i - E_r) \end{bmatrix} = \overline{\mathbb{M}}_{\pm,T} \begin{bmatrix} E_t \\ Y_t E_t \end{bmatrix}.$$

Transmission Matrix

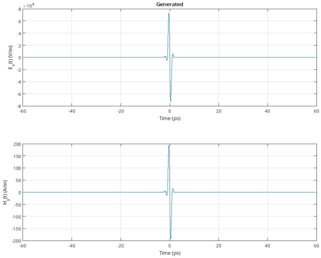
Solving for E_r and $t_\pm$, the two linear equations give:

$$t_\pm = \frac{2Y_i}{(C_\pm + D_\pm Y_t) + Y_i(A_\pm + B_\pm Y_t)}.$$

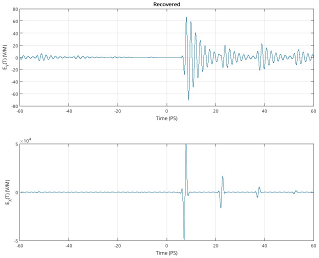
Factor out Y_i^{-1}, giving the field transmission coefficient for the $\hat{\sigma}_\pm$ component is:

$$t_\pm(\nu) = \frac{2}{(A_\pm + Y_t Y_i^{-1} D_\pm) \pm i(Y_i^{-1} C_\pm - Y_t B_\pm)} \tag{11}$$

Our Simulation Result



(a) Generated



(b) Recovered

Figure: Generated and Recovered $E(t)$.

Transfer matrix method for precise determination of thicknesses in a 150-ply polyethylene composite material

Reference: [Palka et al., 2015]

Material

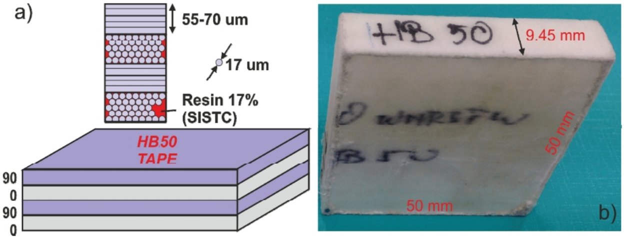


Figure: 154 plies of average thickness $61 \pm 7\mu m$ refractive index $n_c = 1.521 + i0.002$, i.e birefringence $\Delta n = 0.04$.

Result

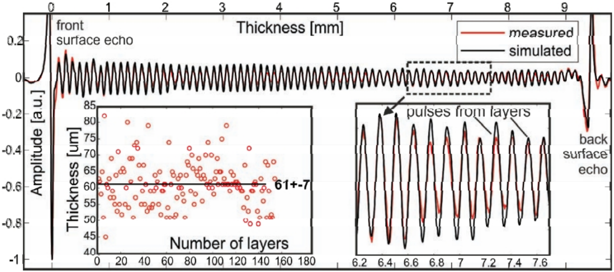


Figure: Waveform reflected from the sample - measured and simulated

The reflection of a terahertz (THz) pulse from a multilayer system is described using transfer matrices. The interface matrix between the i-th and $(i+1)$-th layers:

$$D_{i,j} = \frac{1}{t_{ij}} \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix} \tag{10}$$

where r_{ij} and t_{ij} are the Fresnel coefficients.
The propagation matrix for the i-th layer:

$$P_i(\omega) = \begin{bmatrix} \exp\left(\frac{i\omega N_i d_i}{c}\right) & 0 \\ 0 & \exp\left(-\frac{i\omega N_i d_i}{c}\right) \end{bmatrix} \tag{11}$$

where ω is the angular frequency, N_i is the refractive index, d_i is the thickness, and c is the speed of light.

The total transfer matrix:

$$M_{\text{total}}(\omega) = \prod_{i=0}^{k} P_i(\omega) \cdot D_{i,i+1} = \begin{bmatrix} M_{11}(\omega) & M_{12}(\omega) \\ M_{21}(\omega) & M_{22}(\omega) \end{bmatrix} \tag{12}$$

The reflection coefficient:

$$R(\omega) = \frac{M_{21}(\omega)}{M_{11}(\omega)} \tag{13}$$

The reflected THz signal:

$$E_r(t) = F^{-1}\left[R(\omega) \cdot F[E_0(t)]\right] \tag{14}$$

where F denotes the Fourier transform.
The time-domain fitting error function:

$$QERR(d_1, \ldots, d_k) = \sum_t \left|\text{sign}_{\text{meas}}(t) - \text{sign}_{\text{sim}}(d_1, \ldots, d_k, t)\right| \tag{15}$$

Discussed References

Hilton, D. J. (2012).
Cyclotron resonance spectroscopy in a high mobility two dimensional electron gas using characteristic matrix methods.
Opt. Express, 20(28):29717–29726.

Palka, N., Krimi, S., Ospald, F., Beigang, R., and Miedzinska, D. (2015).
Transfer matrix method for precise determination of thicknesses in a 150-ply polyethylene composite material.
In *2015 40th International Conference on Infrared, Millimeter, and Terahertz waves (IRMMW-THz)*, pages 1–2.

TMM application @ Sapphire and BSTS

Data from Anupama Didi

Recap

~**David J. Hilton**
Only applicable for measuring circular dichroism, not bifringence

Transmission Matrix

$$t_+ \equiv \frac{U_t(z_\ell)}{U_i(z_0)} \tag{10a}$$

$$t_- \equiv \frac{P_t(z_\ell)}{P_i(z_0)} \tag{10b}$$

Boundary Conditions:

- At input: $E(z_0) = E_i + E_r,\ H(z_0) = Y_i(E_i - E_r)$.
- At output: $E(z_\ell) = E_t,\ H(z_\ell) = Y_t E_t$.

Using matrix propagation, the field relations yield:

$$\begin{bmatrix} E_i + E_r \\ Y_i(E_i - E_r) \end{bmatrix} = \overline{\mathbb{M}}_{\pm,T} \begin{bmatrix} E_t \\ Y_t E_t \end{bmatrix}.$$

- **Dielectric tensor:** $\bar{\varepsilon}$ describes anisotropic materials with broken TRS but no birefringence.

$$\bar{\varepsilon}(z,\nu) = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ -\varepsilon_{xy} & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \equiv \begin{bmatrix} \varepsilon_{pp} & 0 & 0 \\ 0 & \varepsilon_{mm} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \tag{2}$$

- $\varepsilon_{pp} = \varepsilon_{xx} + i\varepsilon_{xy}$ and $\varepsilon_{mm} = \varepsilon_{yy} - i\varepsilon_{xy}$.
- $\varepsilon_{pp} = \varepsilon_{mm}$ for isotropic materials with TRS, since, $\varepsilon_{xy} = 0$.

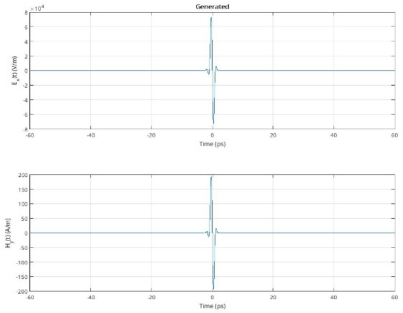
Transmission Matrix

Solving for E_r and $t_\pm$, the two linear equations give:

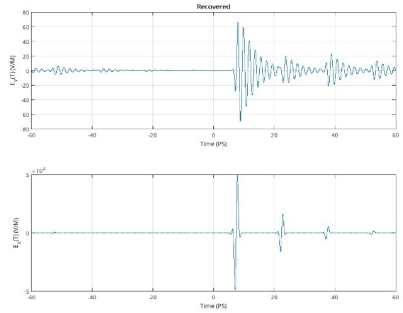
$$t_\pm = \frac{2Y_i}{(C_\pm + D_\pm Y_t) + Y_i(A_\pm + B_\pm Y_t)}.$$

Factor out Y_i^{-1}, giving the field transmission coefficient for the $\hat{\sigma}_\pm$ component is:

$$t_\pm(\nu) = \frac{2}{(A_\pm + Y_t Y_i^{-1} D_\pm) \pm i(Y_i^{-1} C_\pm - Y_t B_\pm)} \tag{11}$$

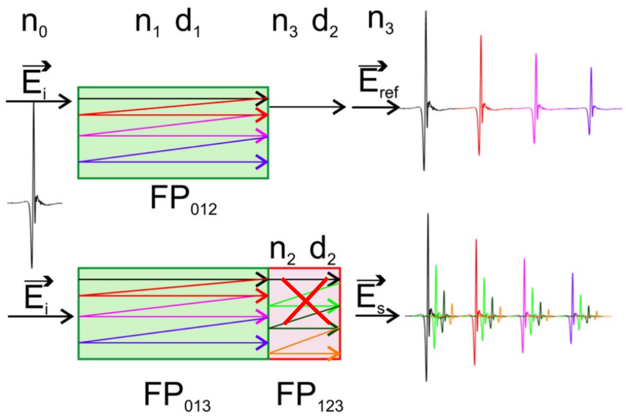


(a) Generated



(b) Recovered

Complex transmission and reflection Fresnel coefficients



General Case

$$t_{jk} = \frac{2n_j}{n_j + n_k} \qquad r_{jk} = \frac{n_j - n_k}{n_j + n_k}.$$

Thin film in between the 2 mediums

Contributions of: $r_{film} \approx 0$, $t_{film} \approx 1$

$$\hat{t}_{1,2} = \frac{2\hat{n}_1}{\hat{n}_1 + \hat{n}_2 + Z_0\hat{\sigma}_{film}},$$
$$\hat{r}_{1,2} = \frac{\hat{n}_1 - \hat{n}_2 - Z_0\hat{\sigma}_{film}}{\hat{n}_1 + \hat{n}_2 + Z_0\hat{\sigma}_{film}},$$

$Z_0 = 376.7$ Ohms is the free space impedance.

Or approximate the film to cause infinite reflections

FP method

$$P_j = \mathrm{e}^{-\mathrm{i}k_0 d_j n_j},$$

$$FP_{jkl} = \sum_{m=0}^{M} (r_{kl} P_k r_{jk} P_k)^m,$$

$$E_\mathrm{s} = t_{01} P_1 t_{12} P_2 t_{23} FP_{012} FP_{123} E_\mathrm{i},$$
$$E_\mathrm{ref} = t_{01} P_1 t_{13} P_3 FP_{013} E_\mathrm{i}.$$

Implies, $$\frac{E_\mathrm{s}}{E_\mathrm{ref}} = \frac{t_{01} P_1 t_{12} P_2 t_{23} FP_{012} FP_{123}}{t_{01} P_1 t_{13} P_3 FP_{013}}$$

For thick substrate, M=0 $FP_{012} = FP_{013} = 1.$

$$\boxed{\frac{E_\mathrm{s}}{E_\mathrm{ref}} = \frac{t_{12} t_{23}}{t_{13}} \frac{P_2}{P_3} FP_{123}.}$$

Tinkham formula

(for a conductor as the thin sample material)

If change in phase and the absorption in the sample layer negligible

$$\mathrm{Re}(n_2)k_0 d_2 \ll 1 \quad \text{and} \quad \mathrm{Im}(n_2)k_0 d_2 \ll 1.$$

$$\boxed{T = \frac{1+n_1}{1+n_1 Z_0 \sigma_2 d_2}}$$

Is a non-trivial Taylor expansion of:

$$\frac{E_s}{E_{ref}} = \frac{t_{12}t_{23}}{t_{13}} \frac{P_2}{P_3} FP_{123}.$$

$$\hat{t}_{1,2} = \frac{2\hat{n}_1}{\hat{n}_1 + \hat{n}_2 + Z_0\hat{\sigma}_{film}},$$
$$\hat{r}_{1,2} = \frac{\hat{n}_1 - \hat{n}_2 - Z_0\hat{\sigma}_{film}}{\hat{n}_1 + \hat{n}_2 + Z_0\hat{\sigma}_{film}},$$

$$FP_{jkl} = \sum_{m=0}^{M=\infty} (r_{kl}P_k r_{jk}P_k)^m = \frac{1}{1 + r_{jk}r_{kl}P_k^2}.$$

(Infinite reflections approximation)

Basic TMM

$$D_{i,j} = \frac{1}{t_{ij}} \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix},$$

$$P_i(\omega) = \begin{bmatrix} \exp\left(\frac{i\omega N_i d_i}{c}\right) & 0 \\ 0 & \exp\left(-\frac{i\omega N_i d_i}{c}\right) \end{bmatrix},$$

$$M_{total}(\omega) = \prod_{i=0}^{k} P_i(\omega) \cdot D_{i,i+1} = \begin{bmatrix} M_{11}(\omega) & M_{12}(\omega) \\ M_{21}(\omega) & M_{22}(\omega) \end{bmatrix}.$$

Reflection Geometry

Transmission

$$\begin{bmatrix} E_{\text{in}} \\ E_{refl} \end{bmatrix} = M_{total}(\omega) \begin{bmatrix} E_{\text{trans}} \\ E_{\text{in2}} \end{bmatrix}$$

$$R(\omega) = \frac{M_{21}(\omega)}{M_{11}(\omega)}.$$

$$T(\omega) = \frac{1}{M_{11}(\omega)}$$

$$E_r(t) = F^{-1}[R(\omega) \cdot F[E_0(t)]].$$

$$E_t(t) = F^{-1}[T(\omega) . F[E_0(t)]]$$

$$R_{total}(\omega) = \frac{R_{sample}(\omega)}{R_{reference}(\omega)}$$

$$T_{total}(\omega) = \frac{T_{sample}(\omega)}{T_{reference}(\omega)}$$

For Simple substrate and air

$$T_{total}(\omega) = \frac{M_{ref,11}}{M_{subs,11}} = \frac{(P_0(d_1,\omega))_{11}}{(D_{01} P_1(d_1,\omega) D_{12})_{11}}$$

$$\hat{t}_{1,2} = \frac{2\hat{n}_1}{\hat{n}_1 + \hat{n}_2 + Z_0\hat{\sigma}_{film}},$$

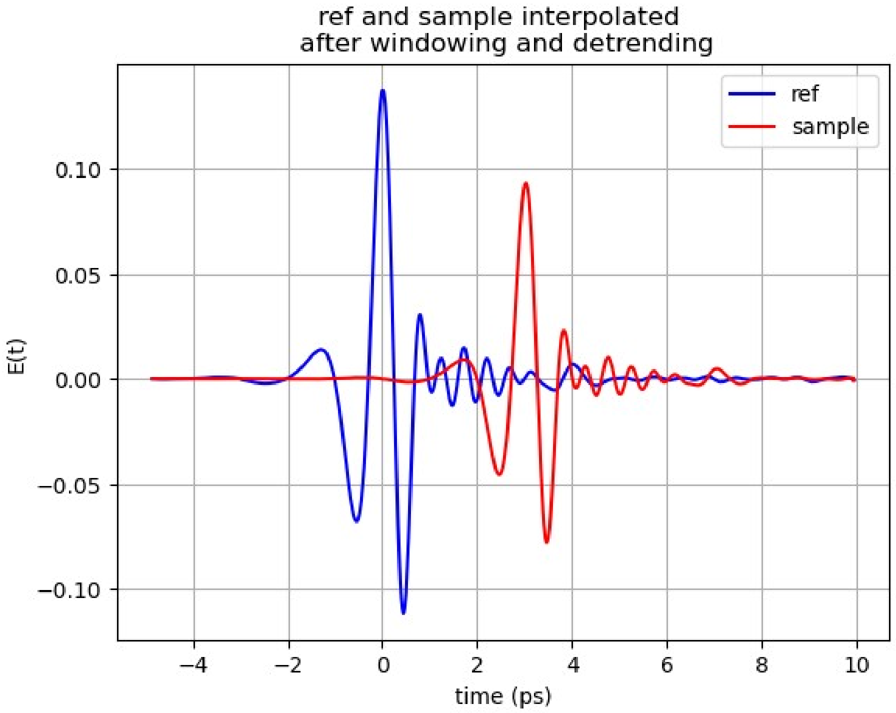
$$\hat{r}_{1,2} = \frac{\hat{n}_1 - \hat{n}_2 - Z_0\hat{\sigma}_{film}}{\hat{n}_1 + \hat{n}_2 + Z_0\hat{\sigma}_{film}},$$

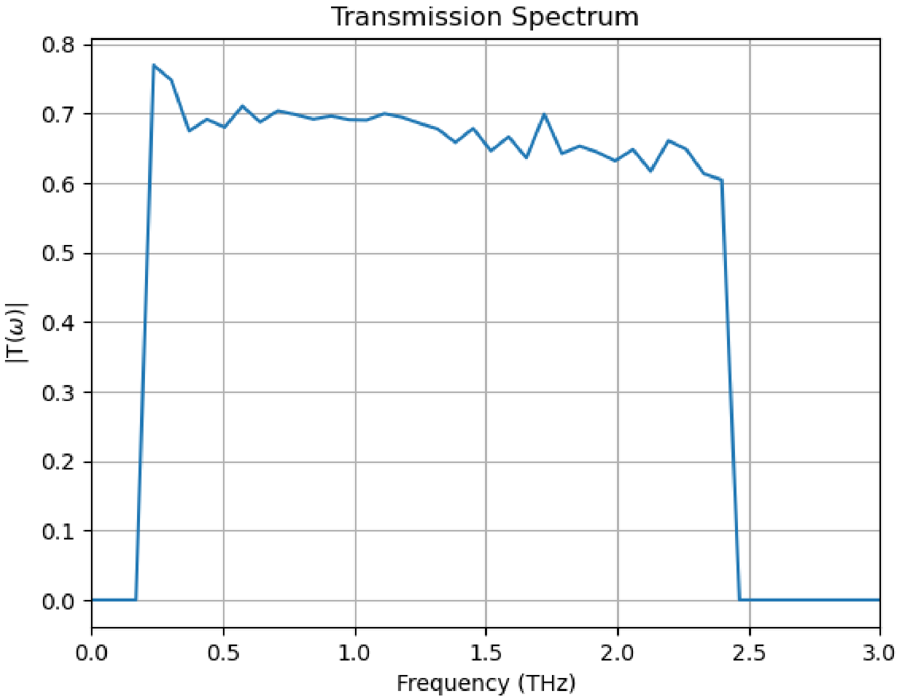
(replace, when trying modified Tinkham)

Results

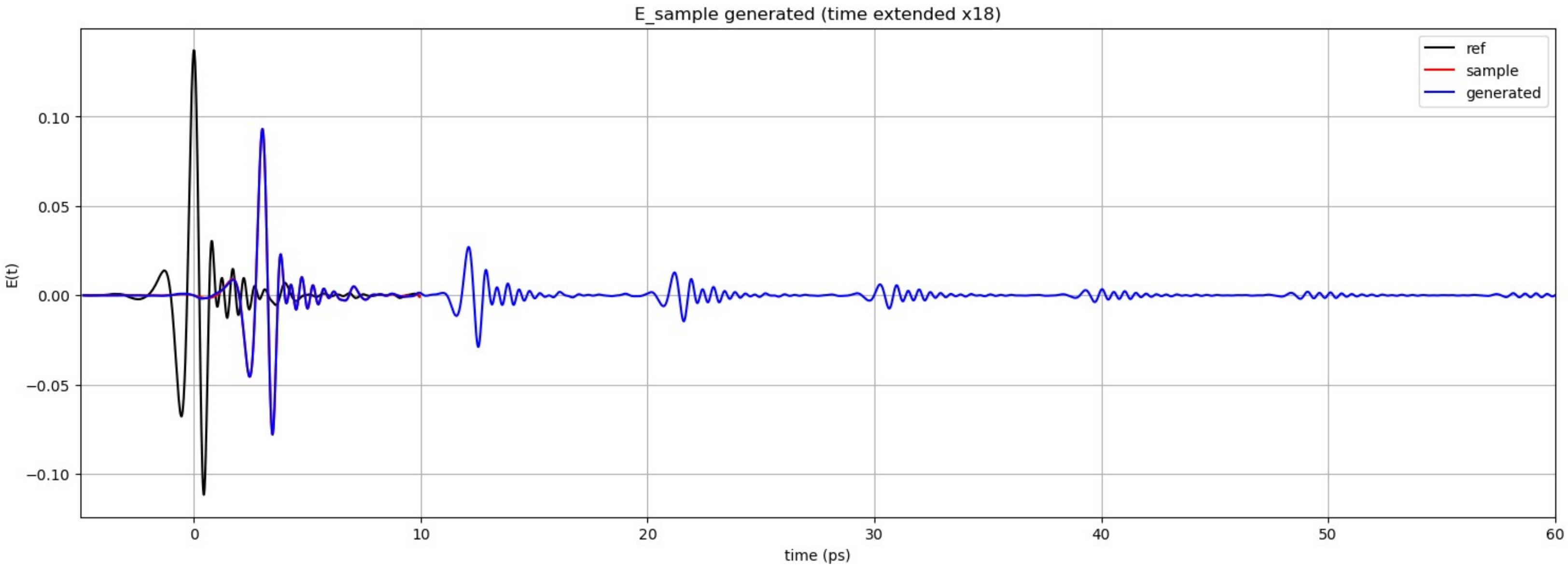
Sapphire only

Given Data


ref and sample interpolated
after windowing and detrending
ref
sample
E(t)
time (ps)
0.10
0.05
0.00
−0.05
−0.10
−4
−2
0
2
4
6
8
10

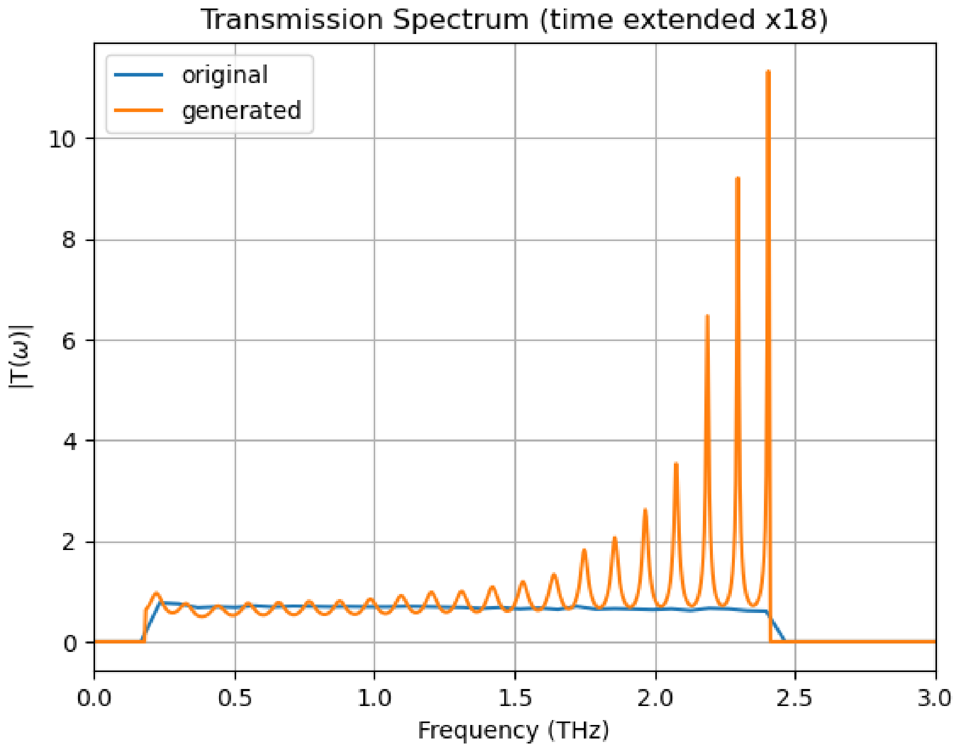

Transmission Spectrum
|T(ω)|
Frequency (THz)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

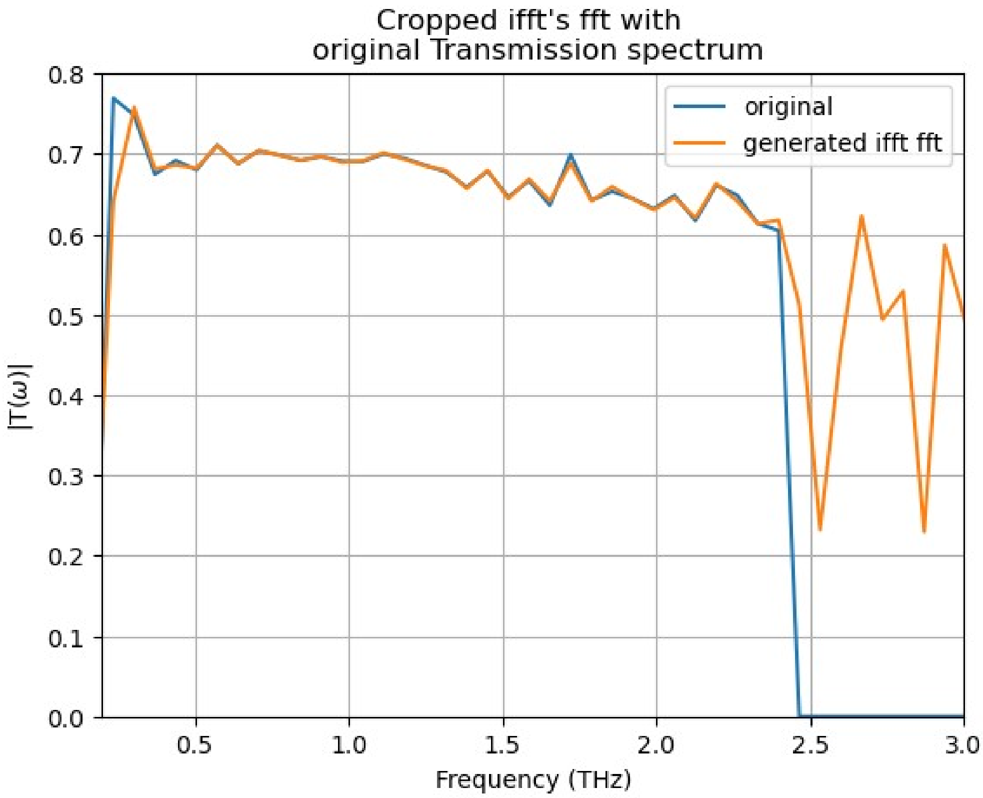
Time domain
Fit with TMM's T(w)'s FFT



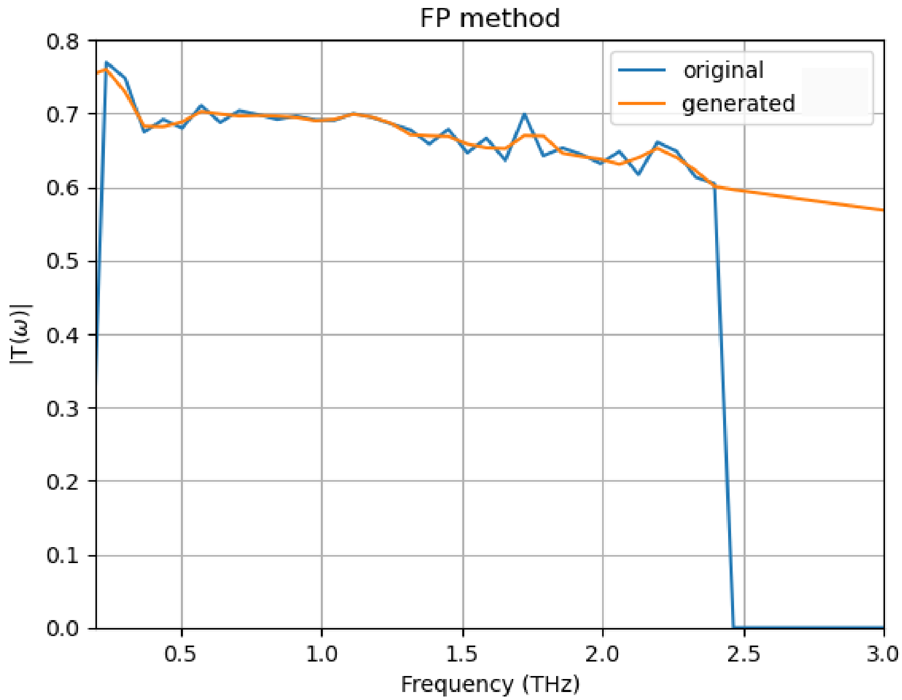
(Ifft fft method)

Time Domain fit's implications

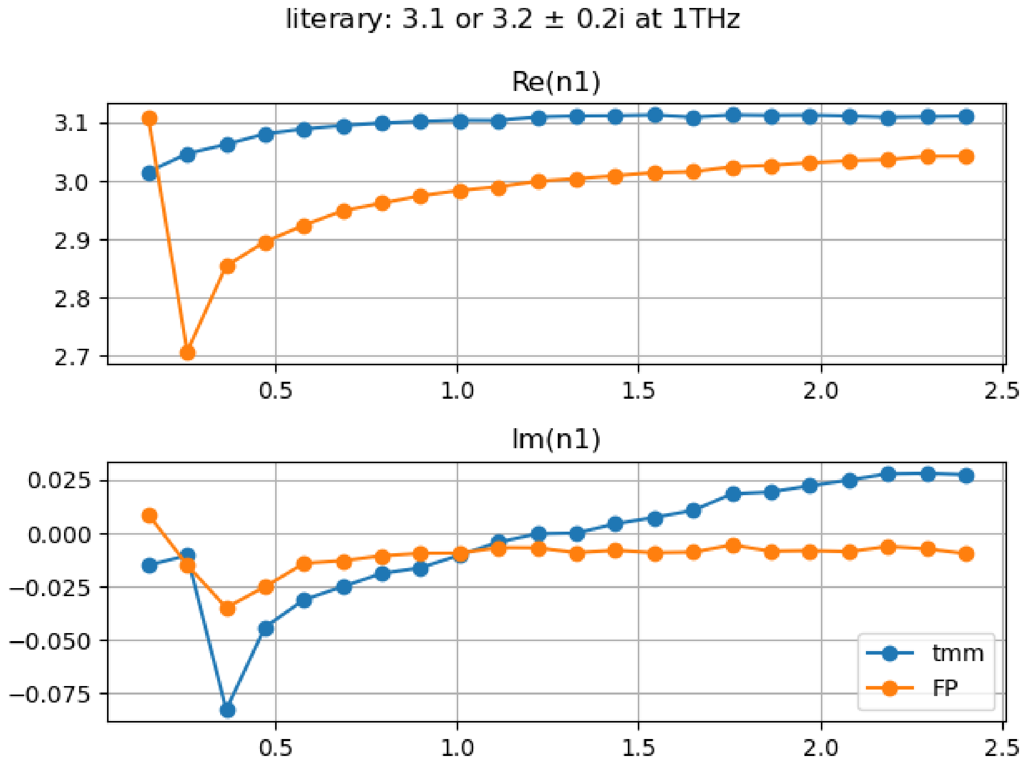




Fit with FP Method's T(w) (freq domain)



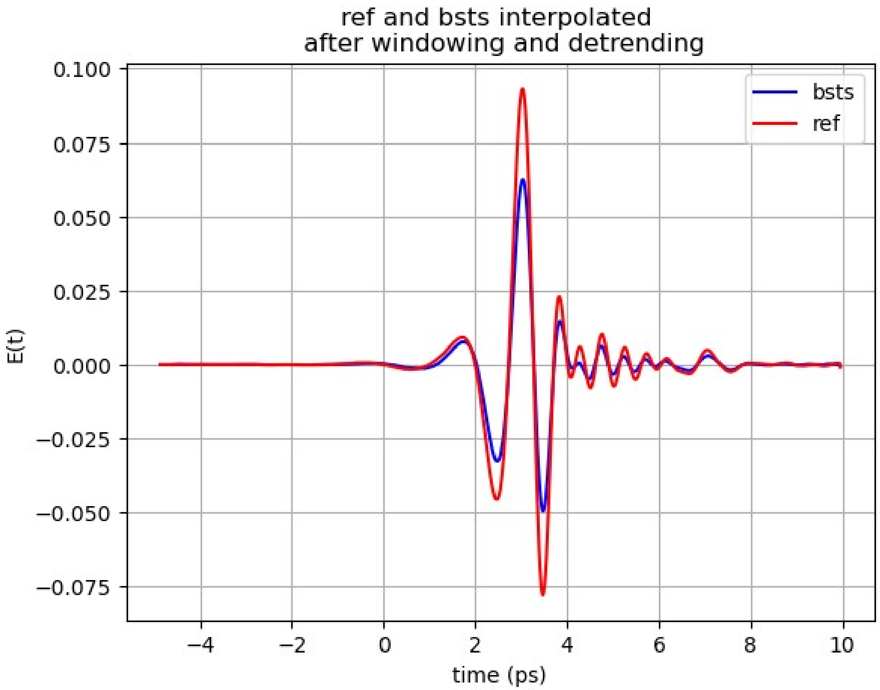
Predicted Complex Refractive Index

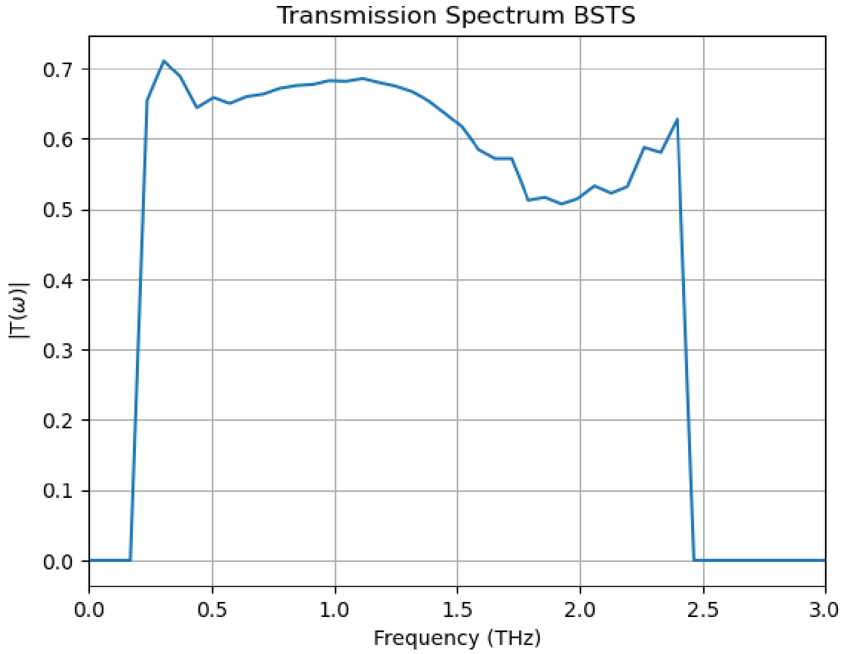


Results

Sapphire + BSTS

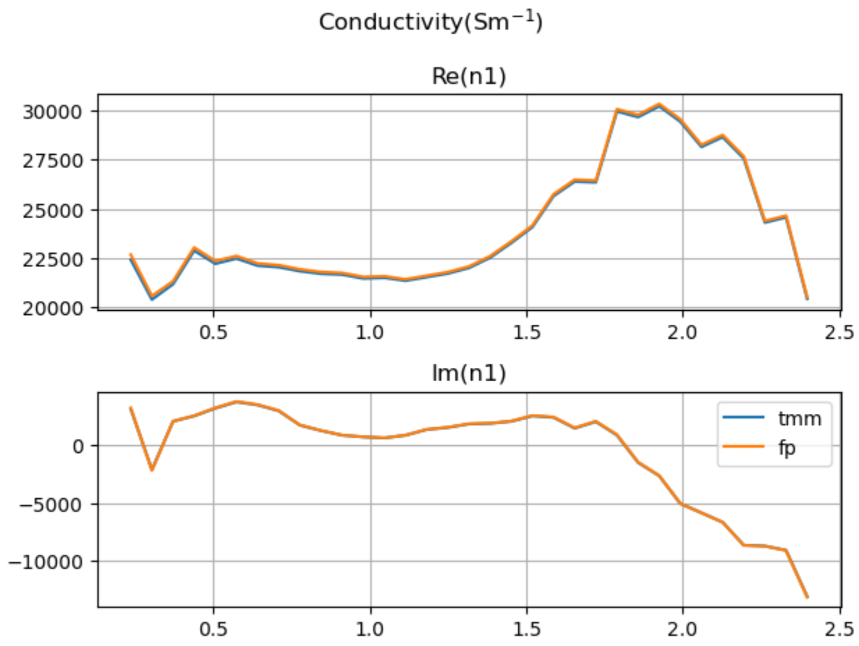
Direct Tinkham on obtained n



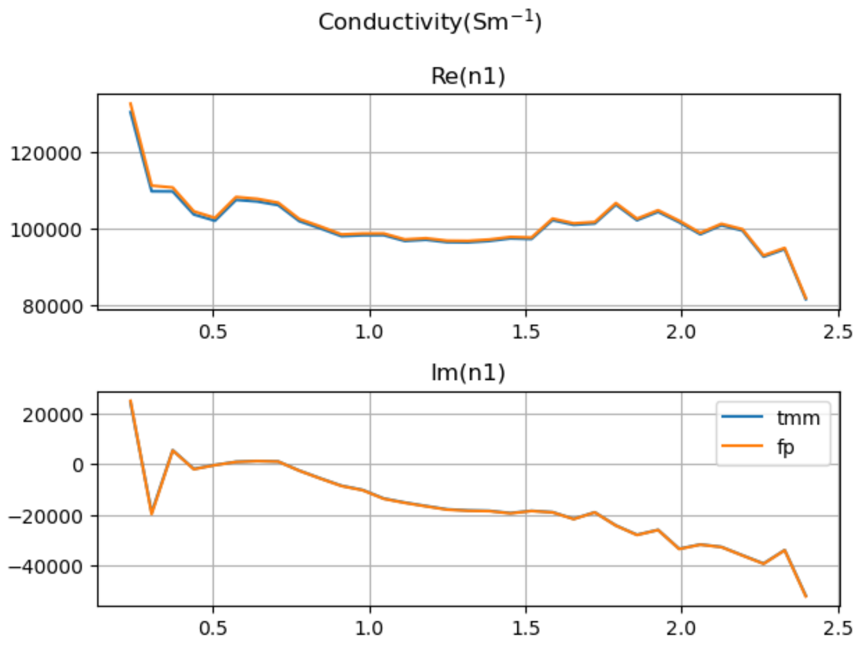


(given data)

Direct Tinkham on obtained n

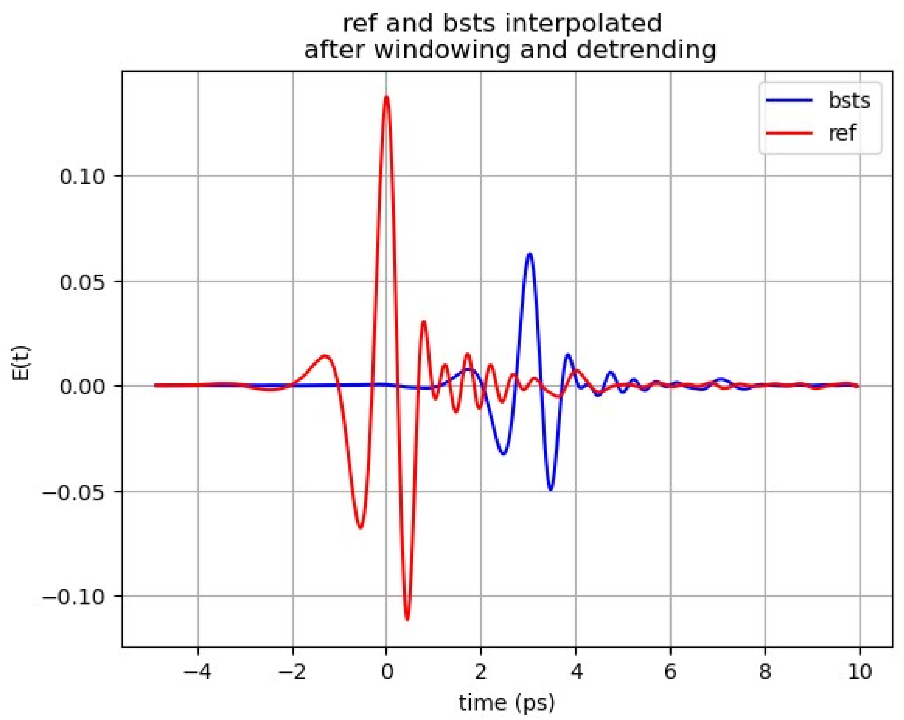


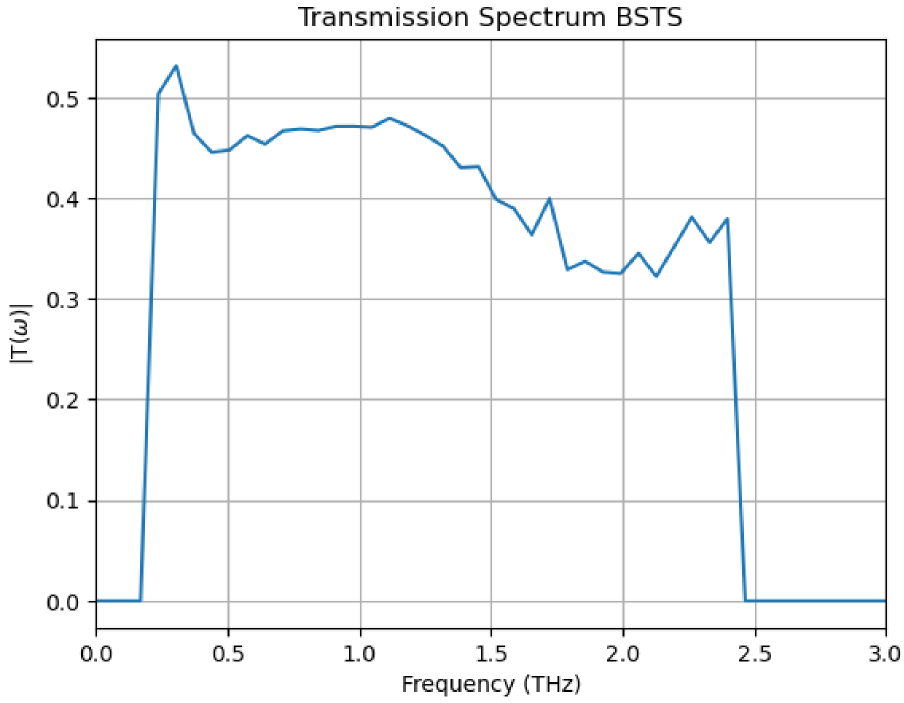
200nm BSTS



50nm BSTS

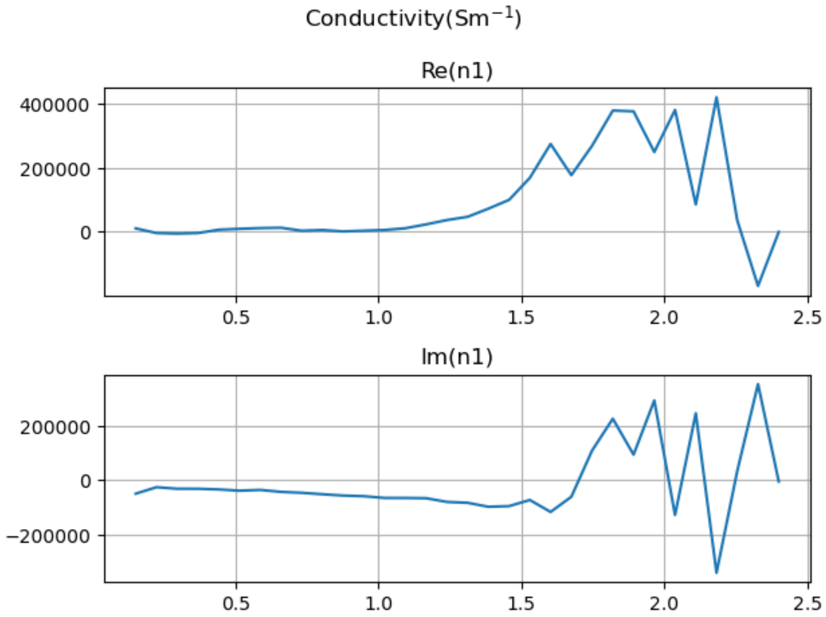
TMM modified

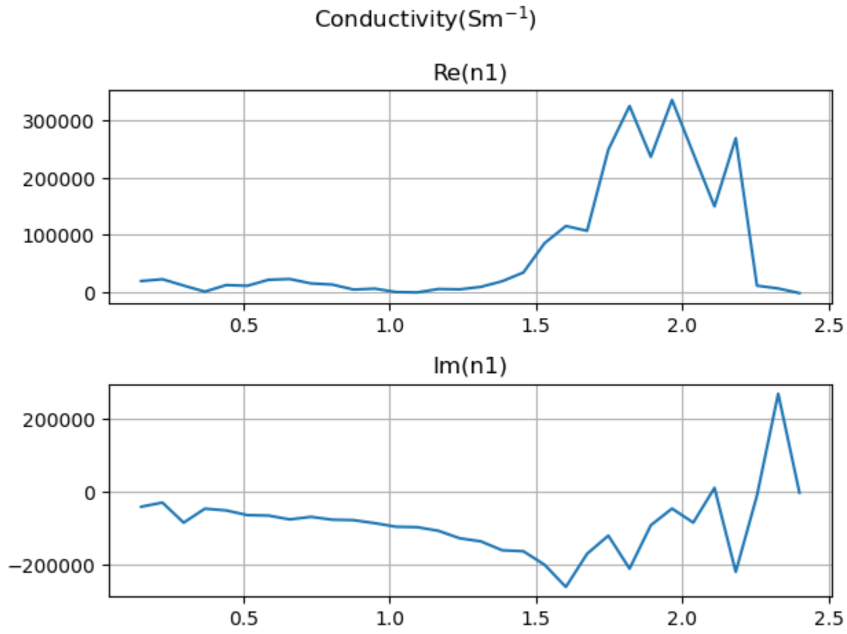


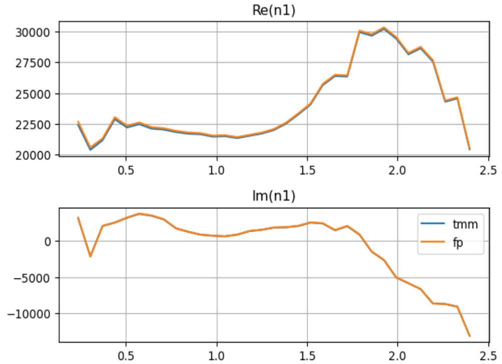


(given data)

TMM modified

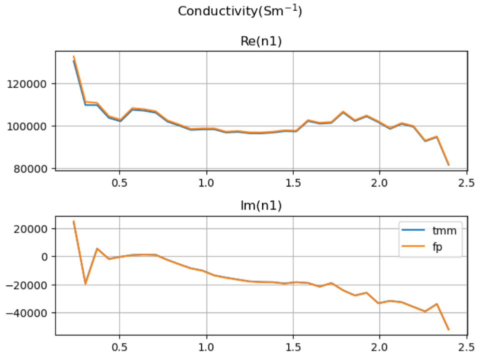






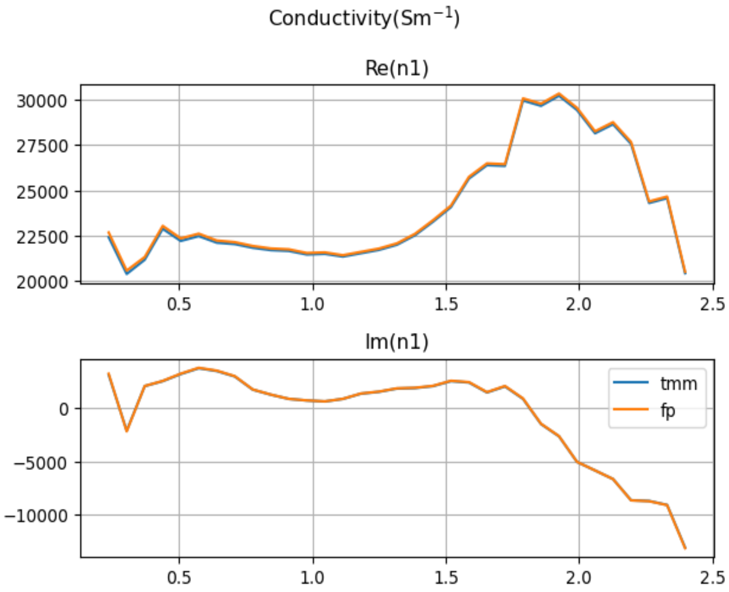
200nm BSTS

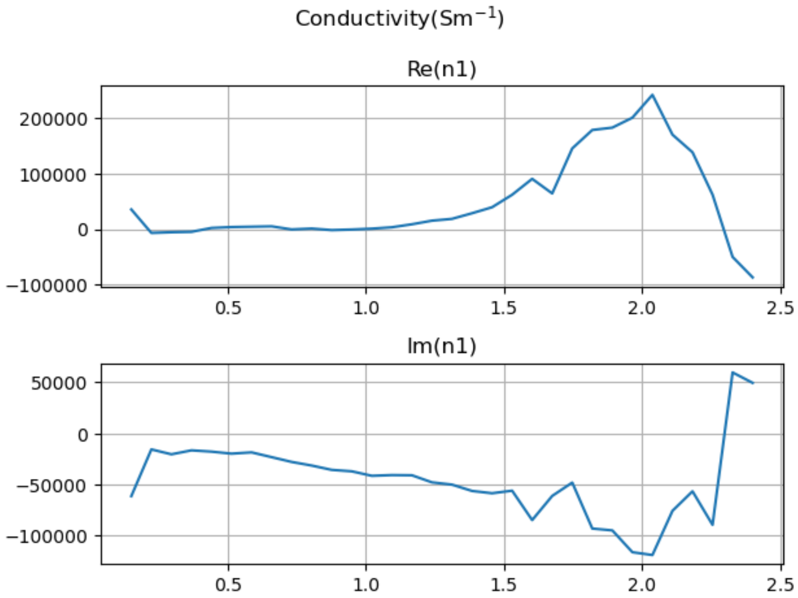
50nm BSTS



D_{23} changed

TMM modified

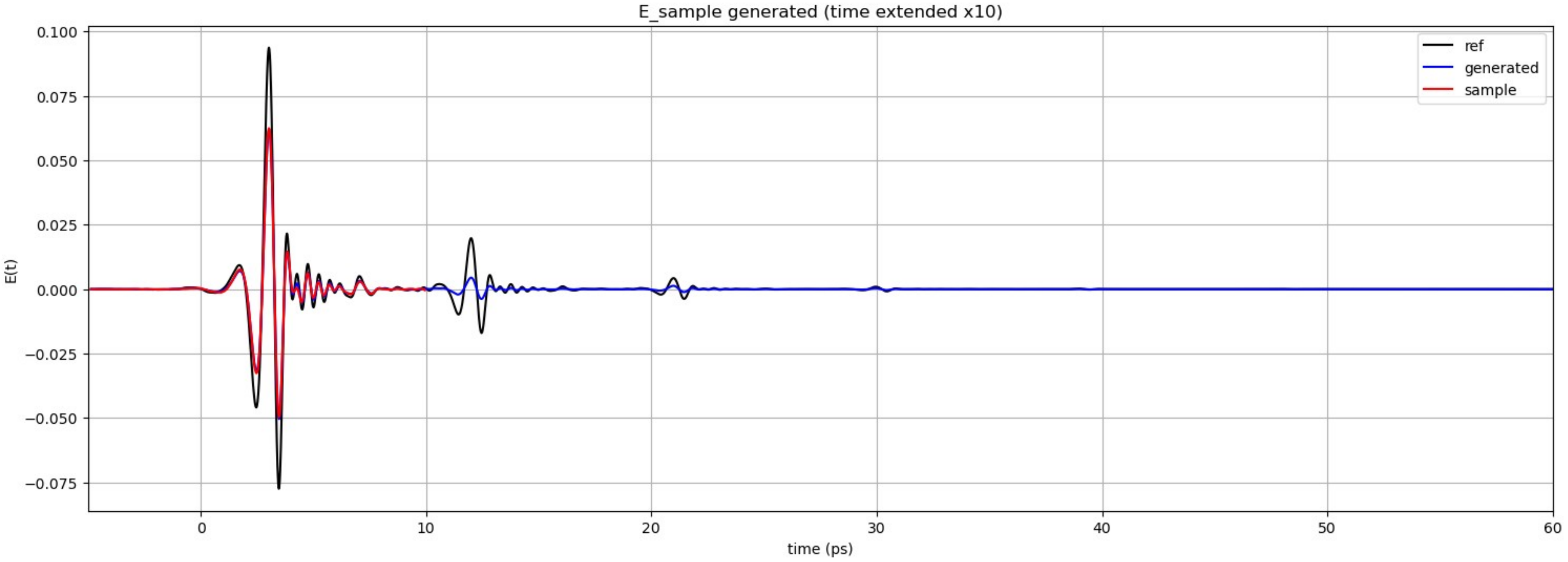




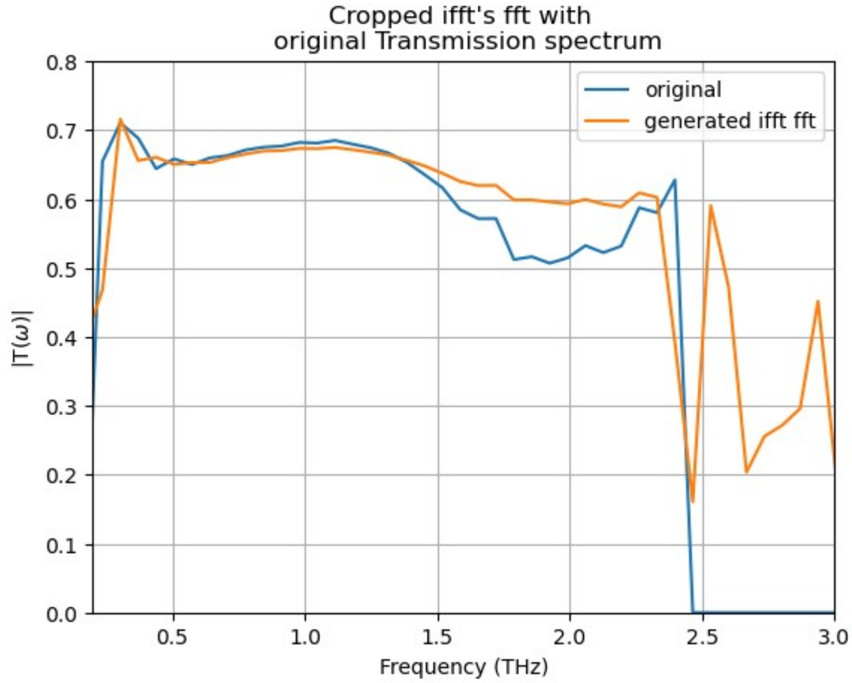
lambda_reg = 1e-3

D_{23} changed(200nm)

Updates on TMM for BSTS



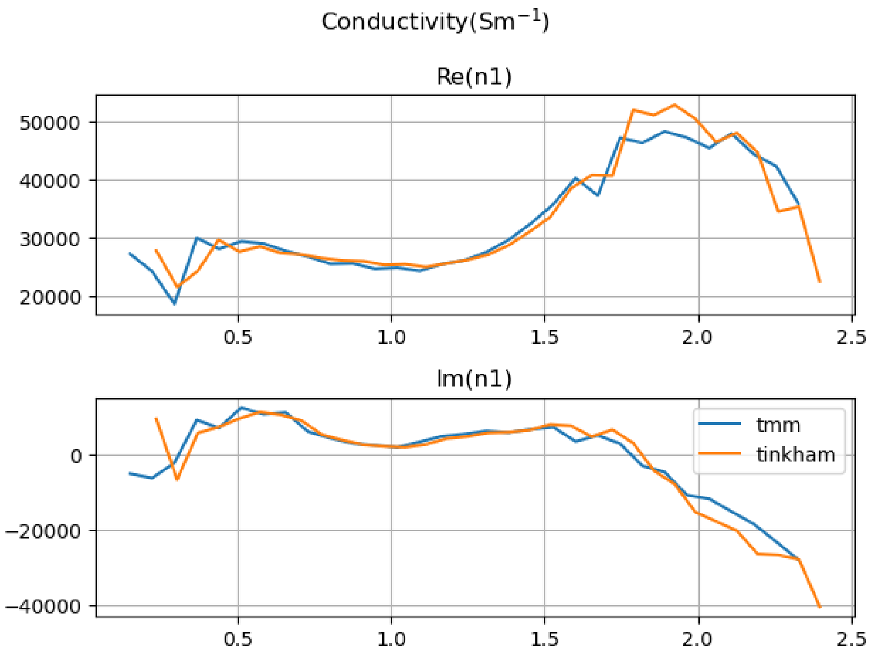
The time-domain fit is behaving slightly incorrectly, causing misalignments around the peak range (1.5–2.3)



This issue can be fixed—it is caused by the fitting-residual definition, which leads to early truncation due to a global tolerance.

Final TMM vs Tinkham

With the slight misfit being there



For 200nm sample with no lambda regression